HEAT KERNEL EXPANSION FOR OPERATORS IN SPACES WITH METRIC INCOMPATIBLE WITH CONNECTION

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Abstract

A method for calculation of the DWSG coefficients for operators in spaces with metric incompatible with connection is suggested based on a generalization of the pseudodifferential operators technique. By using the proposed method, the lowest DWSG coefficients are calculated for minimal operators of the second and fourth order and for nonminimal operators of the type $H^{\mu\nu} = -g^{\mu\nu}g_{\alpha\beta}\nabla^\alpha\nabla^\beta + a\nabla^\mu\nabla^\nu + X^{\mu\nu}$ in spaces with metric incompatible with connection.

1 Introduction

One of the most convenient tools for solution of various mathematical and physical problems dealing with curved manifolds is the so called heat kernel. Usually it is sufficient to know only its asymptotic expansion. The methods to obtain this expansion are numerous and well known for differential operators and manifolds of different kinds [1-3]. The most popular is that of DeWitt [1,4] which suggests a certain ansatz for heat kernel matrix elements. The method possesses the explicit covariance with respect to gauge and general-coordinate transformations. However, the DeWitt technique does not apply to higher-order operators, nonminimal operators, operators whose leading term is not a power of the Laplace operator, and operators defined on spaces with metric incompatible with connection.

Recently, using the Widom generalization [5] of the pseudodifferential operators technique a new algorithm was developed [6] for computing the DeWitt-Seeley-Gilkey (DWSG) coefficients. The method is explicitly gauge and geometrically covariant and allows for carrying out calculations of the DWSG coefficients by computer [7]. As was shown in [8,9,10], the method permits a generalization to the case of Riemann-Cartan manifolds, i.e., manifolds with torsion, and to the case of nonminimal differential operators.

In this work, by using a generalization of the method of [6], we consider the problem of calculation of the DWSG coefficients for operators in spaces with metric incompatible with connection. This problem arises in studying of the different unified scenarious dealing with dynamically generated gravity as well as in the investigating of the effective Lagrangians of the gauge theories and gravity (for more details see the conclusion of this work).

This work is built as follows. In Section 2 we calculate the lowest E_2 DWSG coefficient for minimal operators of the second and fourth order and in Section 3 the lowest E_2 DWSG coefficient for the nonminimal operator $H^{\mu\nu} = -g^{\mu\nu} g_{\alpha\beta} \nabla^\alpha \nabla^\beta + a \nabla^\mu \nabla^\nu + X^{\mu\nu}$.

2 The DWSG coefficients for minimal operators of the second and fourth order

In this section we generalize the method of [6] to the problem of calculation of the DWSG coefficients for minimal operators of the second and fourth

order in spaces with metric incompatible with connection

$$A_1 = -g^{\mu\nu}\nabla_\mu\nabla_\nu + b^\mu\nabla_\mu + X, \quad (1)$$

$$A_2 = g^{\mu\nu}g^{\alpha\beta}\nabla_\mu\nabla_\nu\nabla_\alpha\nabla_\beta + b^{\mu\nu\alpha}\nabla_\mu\nabla_\nu\nabla_\alpha + C^{\mu\nu}\nabla_\mu\nabla_\nu + d^\mu\nabla_{mu} + X, \quad (2)$$

where X is an arbitrary matrix with respect to bundle space indices.

For the sake of completeness, we would like to recall here the most important steps of calculation of the DWSG coefficients in the method proposed in [6]. In fact, the generalization of this method to the case when operators are defined on spaces with metric incompatible with connection is rather straightforward and when describing this method we simply indicate what modifications are needed in the case of operators acting on spaces with metric incompatible with connection.

It is well known [2,3,15] that for a positive elliptic differential operator A of order $2r$ on the n-dimensional manifold M, the diagonal matrix elements of the heat kernel admit the following asymptotic expansion at $t \to 0_+$:

$$< x|e^{-tA}|x > \simeq \sum_m E_m(x|A)t^{(m-n)/2r}, \quad (3)$$

where the summation is carried out over all non-negative integers m. The DWSG coefficients $E_m(x|A)$ reflect the structure of both the operator A and the manifold M. It is the well-established fact that they are local covariant quantities built from the coefficient functions of the operator, curvature, torsion, and their covariant derivatives. In our case this list includes also covariant derivatives of the metric tensor. For the sake of simplicity, in what follows we restrict without loss of generality our consideration to the case $n = 4$.

To calculate the DWSG coefficients, we use the spectral representation of the heat kernel $\exp(-tA)$:

$$e^{-tA} = \int_C \frac{id\lambda}{2\pi} e^{-t\lambda}(A-\lambda)^{-1}, \quad (4)$$

where the contour C goes counterclockwise around the spectrum of the operator A. This reduces our calculations to those for the resolvent $(A-\lambda)^{-1}$. The matrix elements of last one satisfy the following equation:

$$(A(x,\nabla_\mu) - \lambda)\, G(x,x',k;\lambda) = \frac{1}{\sqrt{g}}\delta(x-x'). \quad (5)$$

To solve it, we employ the pseudodifferential operators technique, with the resolvent represented as

$$G(x,x';\lambda) = \int \frac{d^4k}{(2\pi)^4\sqrt{g(x')}}\, e^{il(x,x',k)}\sigma(x,x',k;\lambda), \tag{6}$$

and

$$\frac{\delta(x-x')}{\sqrt{g}} = \int \frac{d^4k}{(2\pi)^4\sqrt{g(x')}}\, e^{il(x,x',k)}I(x,x',k;\lambda). \tag{7}$$

Here $l(x,x',k)$ is the phase function, $I(x,x')$ - the so called function of parallel transport, and $\sigma(x,x',k;\lambda)$ - the amplitude [14, 15]. In the flat space $l(x,x')$ is nothing else as $l(x,x',k) = k_\mu(x-x')^\mu$ and in the case of more general manifolds, it is a real function, biscalar with respect to general-coordinate transformations. The linearity condition in x is generalized to the requirement for the m*th* symmetrized covariant derivative of l to vanish as $x \to x'$:

$$\begin{aligned} \{\nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_m}\}\, l|_{x=x'} = [\{\nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_m}\}\, l] = \\ k_{\mu_1} \text{ for } m=1 \text{ and } 0 \text{ for } m\neq 1. \end{aligned} \tag{8}$$

In Eq. (7) the curly brackets denote symmetrization in all indices and the square brackets mean that the coincidence limit $x \to x'$ is taken. The local properties of the function l are sufficient when obtaining the diagonal heat kernel expansion. The biscalar function $I(x,x')$ is defined analogously:

$$\begin{aligned} [\mathrm{I}] &= 1, \\ [\{\nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_m}\}\,\mathrm{I}] &= 0 \;\; m\geq 1, \end{aligned} \tag{9}$$

the unity in Eq. (8) is generally a matrix unity.

We first consider the case of the second order operator A_1 in Eq. (1). It follows from Eq. (4) that the amplitude σ satisfies the equation

$$\begin{aligned} (-g^{\mu\nu}(\nabla_\mu + i\nabla_\mu l)(\nabla_\nu + i\nabla_\nu l) + b^\mu(\nabla_\mu + i\nabla_\mu l) \\ + X - \lambda)\sigma(x,x',k;\lambda) = \mathrm{I}(x,x'). \end{aligned} \tag{10}$$

To generate expansion (2), we introduce an auxiliary parameter ϵ into Eq. (9) according to the rule $l \to l/\epsilon$, $\lambda \to \lambda/\epsilon^2$, and expand the amplitude into a formal series in powers of ϵ

$$\sigma(x,x',k;\lambda) = \sum_{m=0}^{\infty} \epsilon^{2+m}\sigma_m(x,x',k;\lambda) \tag{11}$$

(the parameter ϵ is then set equal to one).

In such a way Eq. (10) gives us the recursion equations to determine the coefficients σ_m from, and, eventually, this procedure leads to expansion (2) where the DWSG coefficients $E_m(x|A)$ are expressed through the integrals of $[\sigma_m]$ in the form [6]:

$$E_m(x|A) = \int \frac{d^n k}{(2\pi)^n \sqrt{g}} \int_C \frac{id\lambda}{2\pi} e^{-\lambda} [\sigma_m](x, x, k; \lambda). \tag{12}$$

In the case of the second order operator A_1 the recursion equations for the lowest σ_0, σ_1, and σ_2 coefficients take the form

$$(g^{\mu\nu}\nabla_\mu l \nabla_\nu l - \lambda)\sigma_0 = I,$$

$$(g^{\mu\nu}\nabla_\mu l \nabla_\nu l - \lambda)\sigma_1 + (-2ig^\mu\nu\nabla_\mu l \nabla_\nu - ig^{\mu\nu}\nabla_{\mu\nu} l + ib^\mu \nabla_\nu l)\sigma_0 = 0,$$

$$(g^{\mu\nu}\nabla_\mu l \nabla_\nu l - \lambda)\sigma_2 + (-2ig^\mu\nu\nabla_\mu l \nabla_\nu - ig^{\mu\nu}\nabla_{\mu\nu} l + ib^\mu \nabla_\nu l)\sigma_1 +$$
$$(-g^{\mu\nu}\nabla_\mu \nabla_\nu + b^\mu \nabla_\mu + X)\sigma_0 = 0. \tag{13}$$

Hence, we find

$$\sigma_0 = \frac{I}{\nabla^\mu l \nabla_\mu l - \lambda},$$

$$\sigma_1 = \sigma_0(2ig^\mu\nu\nabla_\mu l \nabla_\nu + ig^{\mu\nu}\nabla_{\mu\nu} l - ib^\mu \nabla_\nu l)\sigma_0,$$

$$\sigma_2 = \sigma_0(2ig^\mu\nu\nabla_\mu l \nabla_\nu + ig^{\mu\nu}\nabla_{\mu\nu} l - ib^\mu \nabla_\nu l)\sigma_1 +$$
$$\sigma_0(g^{\mu\nu}\nabla_\mu \nabla_\nu - b^\mu \nabla_\mu - X)\sigma_0. \tag{14}$$

Perfoming direct algebraic calculations, from (14) and (7) we get the lowest E_2 coefficient for the operator A_1

$$E_2 = \frac{1}{(4\pi)^2}\Big(\frac{R}{6} - X + \nabla_\mu b^\mu - \frac{b^2}{4} - \frac{g_{\alpha\beta}\nabla^\mu\nabla_\mu g^{\alpha\beta}}{12}\frac{g^{\nu\beta}\nabla_\mu\nabla_\nu g^{\mu\beta}}{3} +$$

$$\frac{g_{\mu\nu}g_{\alpha\beta}\nabla^\kappa g^{\mu\nu}\nabla_\kappa g^{\alpha\beta}}{48}+\frac{g_{\mu\alpha}g_{\nu\beta}\nabla^\kappa g^{\mu\nu}\nabla_\kappa g^{\alpha\beta}}{24}-\frac{g_{\alpha\beta}\nabla_\mu g^{\mu\nu}\nabla_\nu g^{\alpha\beta}}{12}+$$

$$\frac{g_{\mu\beta}\nabla_\alpha g^{\mu\nu}\nabla_\nu g^{\alpha\beta}}{12}-\frac{g_{\nu\beta}\nabla_\mu g^{\mu\nu}\nabla_\alpha g^{\alpha\beta}}{4}), \tag{15}$$

where we use (see [6])

$$\int \frac{d^n k}{(2\pi)^n\sqrt{g}}k_{\mu_1}k_{\mu_2}\dots k_{\mu_{2s}}f(k^2)=$$

$$g_{(\mu_1\mu_2\dots\mu_{2s})}\frac{1}{(4\pi)^{n/2}2^s\Gamma(n/2+s)}\int_0^\infty dk^2(k^2)^{(n-2)/2+s}f(k^2), \tag{16}$$

and $g_{(\mu_1\mu_2\dots\mu_{2s})}$ is the symmetrized sum of metric tensor products. Evidently, for the space with metric compatible with connection, i.e., if $\nabla^\mu g_{\alpha\beta}=0$ we restore the well-known result $E_2=\frac{1}{(4\pi)^2}(\frac{R}{6}-X)$.

For the operators of the fourth order A_2, the equation for the amplitude takes the similar but some more cumbersome form

$$(g^{\mu\nu}g^{\alpha\beta}(\nabla_\mu+i\nabla_\mu l)(\nabla_\nu+i\nabla_\nu l)(\nabla_\alpha+i\nabla_\alpha l)(\nabla_\beta+$$

$$i\nabla_\beta l)+b^{\mu\nu\alpha}(\nabla_\mu+i\nabla_\mu l)(\nabla_\nu+i\nabla_\nu l)(\nabla_\alpha+i\nabla_\alpha l)+$$

$$C^{\mu\nu}(\nabla_\mu+i\nabla_\mu l)(\nabla_\nu+i\nabla_\nu l)+d^\mu\nabla_\mu+X-\lambda)\sigma=I. \tag{17}$$

Once again, to generate expansion (2) we introduce an auxiliary parameter ϵ into Eq. (19) according to the rule $l\to l/\epsilon$, $\lambda\to\lambda/\epsilon^4$, and expand the amplitude into a formal series in powers of ϵ

$$\sigma(x,x',k;\lambda)=\sum_{m=0}^{\infty}\epsilon^{4+m}\sigma_m(x,x',k;\lambda). \tag{18}$$

Similarly to the case of the operator A_1, we find the lowest E_2 DWSG coefficient for the operator A_2

$$E_2=\frac{\sqrt{\pi}}{(4\pi)^2}\frac{1}{2}(\frac{R}{6}+\frac{C^\mu_\mu}{8}-\frac{9b_\mu\mu\nu b^\alpha_{\nu\alpha}+6b^{\mu\nu\alpha}b_{\mu\nu\alpha}}{1024}-$$

$$\frac{3g_{\alpha\beta}\nabla_\mu b^{\mu\nu\alpha}}{32}+\frac{15b^\alpha_{\nu\alpha}\nabla_\mu g^{\mu\nu}}{128}+3g_{\alpha\beta}b^{\mu\nu}_\nu\nabla_\mu g^{\alpha\beta}+$$

$$\frac{6b^{\mu}_{\nu\alpha}\nabla_{\mu}g^{\nu\alpha}}{256}+5g_{\rho\kappa}\nabla^{\mu}g^{\rho\kappa}g_{\alpha\beta}\nabla_{\mu}g^{\alpha\beta}+\frac{10g_{\alpha\rho}g_{\beta\kappa}\nabla_{\mu}g^{\alpha\beta}\nabla^{\mu}g^{\rho\kappa}}{768}-$$

$$\frac{41g_{\alpha\beta}\nabla_{\mu}g^{\mu\nu}\nabla_{\nu}g^{\alpha\beta}}{192}-\frac{g_{\beta\kappa}\nabla_{\mu}g^{\alpha\beta}\nabla_{\alpha}g^{\mu\kappa}}{192}-\frac{17g_{\nu\beta}\nabla_{\mu}g^{\mu\nu}\nabla_{\alpha}g^{\alpha\beta}}{64}$$

$$-\frac{g^{\mu\nu}g_{\alpha\beta}\nabla_{\mu}\nabla_{\nu}g^{\alpha\beta}}{24}+\frac{7\nabla_{\mu}\nabla_{\nu}g^{\mu\nu}}{24}). \qquad (19)$$

3 The DWSG coefficients for the nonminimal operator

In this section we calculate the lowest DWSG coefficient for the nonminimal operator $H^{\mu\nu}=-g^{\mu\nu}g_{\alpha\beta}\nabla^{\alpha}\nabla^{\beta}+a\nabla^{\mu}\nabla^{\nu}+X^{\mu\nu}$ in space with metric incompatible with connection. Operators of this type arise naturally under the quantization of gauge and gravitational fields in arbitrary gauges [11]. In [9] the lowest DWSG coefficients were calculated by using a generalization of the pseudodifferential operators technique. Here, we calculate the lowest E_2 coefficient for the operator $H^{\mu\nu}$ in space with metric incompatible with connection.

The most essential point as compared to the case of minimal operators consists in alteration of recursion relations for the amplitude of resolvent $(H-\lambda)^{-1}$ (see [9]). The equation for the amplitude $\sigma_{\rho\nu}(x,x',k;\lambda)$ has the form

$$(g^{\mu\rho}(\nabla^{\kappa}l\nabla_{\kappa}l-i\nabla^{\kappa}\nabla_{\kappa}l-2i\nabla^{\kappa}l\nabla_{\kappa}-\nabla^{\mu}\nabla_{\mu}-\lambda)+a(i\nabla^{\mu}\nabla^{\rho}l$$
$$-\nabla^{\mu}l\nabla^{\rho}l+i\nabla^{\mu}l\nabla^{\rho}+i\nabla^{\rho}l\nabla^{\mu}+\nabla^{\mu}\nabla^{\rho})+X^{\mu\rho})\sigma_{\rho\nu}=I^{\mu}_{\nu}(x,x'). \qquad (20)$$

Setting $l\to l/\epsilon$, $\lambda\to\lambda/\epsilon^2$, and $\sigma(x,x',k;\lambda)=\sum_{m=0}^{\infty}\epsilon^{2+m}\sigma_m(x,x',k;\lambda)$, we get from Eq. (25) the recursion relations for the coefficients $\sigma_{m\rho\nu}$

$$D^{\mu\rho}\sigma_{0\rho\nu}=I^{\mu}_{\nu}$$

$$D^{\mu\rho}\sigma_{1\rho\nu}+i(-g^{\mu\nu}(\nabla_{\kappa}\nabla^{\kappa}l+2\nabla^{\kappa}l\nabla_{\kappa})+a(\nabla^{\mu}\nabla^{\rho}l+$$

$$\nabla^{\mu}l\nabla^{\rho}+\nabla^{\rho}l\nabla^{\mu}))\sigma_{0\rho\nu}=0$$

$$D^{\mu\rho}\sigma_{m\rho\nu} + i(-g^{\mu\nu}(\nabla_\kappa\nabla^\kappa l + 2\nabla^\kappa l\nabla_\kappa) + a(\nabla^\mu\nabla^\rho l + \nabla^\mu l\nabla^\rho +$$

$$\nabla^\rho l\nabla^\mu))\sigma_{m-1\rho\nu} + (-g^{\mu\rho}g_{\alpha\beta}\nabla^\alpha\nabla^\beta + a\nabla^\mu\nabla^\rho + X^{\mu\rho})\sigma_{m-2\rho\nu} = 0, m \geq 2. \quad (21)$$

The main difference from the case of minimal operators is that for obtaining $\sigma_{m\rho\nu}$ we must now invert the matrix $D^{\mu\rho}$ and differentiate it. Of course, this increases the algerbaic labour bot does not cause essential diffficulties. Solving Eq. (27) and using the formula for the integration in k and λ (see [9])

$$\int \frac{d^n k}{(2\pi)^n\sqrt{g}}(k^2)^p k_{\mu_1}k_{\mu_2}\ldots k_{\mu_{2s}} \int_C \frac{id\lambda}{2\pi}\frac{e^{-\lambda}}{(k^2-\lambda)^q((1-a)k^2-\lambda)^m} =$$

$$g_{(\mu_1\mu_2\ldots\mu_{2s})}\frac{\Gamma(n/2+s+p)F(m,p+s+n/2,q+m;a)}{(4\pi)^{n/2}2^s\Gamma(n/2+s)\Gamma(l+m)}, \quad (22)$$

where $F(a,b,c;z)$ is the Gauss hypergeometric function, we obtain the lowest E_2 DWSG coefficient

$$E_{2\mu\nu} = \frac{1}{(4\pi)^2}(g_{\mu\nu}R(\frac{1}{6} + \frac{a^3-3a^2+2a}{24(1-a)^3}) + R_{\mu\nu}\frac{9a^3-2a^2+12a}{36(1-a)^3} +$$

$$W_{\mu\nu}\frac{a(2a-1)}{2(1-a)^2} - (X_{\mu\nu}-X_{\nu\mu})(\frac{1}{2}+\frac{a}{4(1-a)}) - g_{\mu\nu}X^\alpha_\alpha\frac{a^2}{24(1-a)^2} +$$

$$\frac{g_{\mu\nu}g\kappa\rho T^{\kappa\rho}_{\alpha\beta}}{6(1-a)} + \frac{g_{\mu\nu}T^{\kappa\rho}_{\kappa\rho}(a^2+4)}{3(1-a)} - \frac{g_{\mu\rho}T^{\kappa\rho}_{\alpha\beta}(5a^2+2a+1)}{6(1-a)} -$$

$$\frac{g_{\mu\rho}T^{\kappa\rho}_{\kappa\beta}(2a^2+a)}{1-a} + \frac{g_{\kappa\nu}T^{\kappa\rho}_{\mu\rho}(a^2+4a-1)}{3(1-a)} + \frac{g_{\kappa\rho}T^{\kappa\rho}_{\mu\nu}(a^2+3a-3)}{3(1-a)} +$$

$$\frac{g_{\mu\kappa}g_{\rho\nu}g^{\alpha\beta}T^{\kappa\rho}_{\alpha\beta}(-5a^3+8a^2-a+2)a}{6(1-a)^2} +$$

$$\frac{g_{\mu\kappa}T^{\kappa\rho}_{\rho\nu}a(-2a^3+2a^2+14a-13)}{6(1-a)^2} +$$

$$\frac{g_{\rho\nu}T^{\kappa\rho}_{\mu\kappa}(a^2+2)}{3(1-a)}+\frac{g_{\mu\rho}T^{\kappa\rho}_{\kappa\nu}a(2-a)}{1-a}+$$

$$g^{\alpha\beta}Q^{\kappa}_{\kappa\mu}Q_{\nu\alpha\beta}(-\frac{a}{3}+\frac{15a^3+2a(a-2)(5a^2+45a+26)}{30(1-a)^2})+$$

$$Q^{\kappa}_{\mu\nu}Q^{\rho}_{\rho\kappa}(4+\frac{6a}{1-a}-\frac{a(a-2)(a^2+7a-7)}{6(1-a)^2}+$$

$$\frac{a(a-2)(5a^2+45a+26)}{15(1-a)^2}+$$

$$Q^{\kappa}_{\kappa\mu}Q^{\rho}_{\rho\nu}(-\frac{a(a-2)}{6}+\frac{72a-4a^3-a(a-2)(a+14)-16(6a^2+a+4)}{12(1-a)}+$$

$$\frac{15a^3+a(a-2)(5a^2+45a+26)}{15(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\alpha\beta}Q_{\kappa\mu\nu}(\frac{8a+3}{3(1-a)}-\frac{a(a-2)(5a^2+45a+26)}{30(1-a)^2})+$$

$$Q^{\kappa}_{\mu\nu}Q^{\rho}_{\rho\kappa}(4+\frac{6a}{1-a}-\frac{a(a-2)(a^2+7a-7)}{6(1-a)^2}+$$

$$\frac{a(a-2)(5a^2+45a+26)}{15(1-a)^2}+\frac{a^2}{3(1-a)}+\frac{a^3}{2(1-a)^2}-\frac{a^2}{2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\kappa\beta}Q_{\nu\mu\alpha}(\frac{a(2a^2+a+32)}{6(1-a)}+\frac{15a^3+2a(a-2)(5a^2+45a+26)}{30(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\alpha\beta}Q_{\nu\mu\kappa}(\frac{a(2a^2-5a+10)}{12(1-a)}+\frac{a(a-2)25a^2+185a+104}{120(1-a)^2})+$$

$$Q^{\kappa}_{\mu\rho}Q^{\rho}_{\nu\kappa}(2a+\frac{a(a-2)(85a^2+465a+68)}{120(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\mu\alpha}Q_{\nu\beta\kappa}(-\frac{2a}{3}+\frac{a(2a^2-a+32)}{6(1-a)}+\frac{a(a-2)(15a^2+185a+108)}{60(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\kappa\nu}Q_{\mu\alpha\beta}(2a^2-\frac{a(a+2)}{12}-\frac{a}{3}-$$

$$\frac{a(19a^2+65a+30)}{24(1-a)}-\frac{a(5a^3+35a^2+73a+14)}{60(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\kappa\alpha}Q_{\mu\nu\beta}(\frac{a(3a^2+16a+32)}{12(1-a)}+\frac{a(75a^2-138a-64)}{30(1-a)^2})$$

$$g^{\alpha\beta}Q^{\kappa}_{\alpha\beta}Q_{\mu\nu\kappa}(\frac{a(21a^2+46a-32)}{24(1-a)}+$$

$$\frac{5a^2(a^2-5a-14))+a(a-2)(10a^2+120a+74)}{120(1-a)^2})+$$

$$g^{\alpha\beta}Q^{\kappa}_{\nu\beta}Q_{\mu\alpha\kappa}(\frac{a(a-2)}{3}+\frac{a(16a^2+11a+26)}{12(1-a)}+$$

$$\frac{5a^2(a^2-3a-8)+2a(a-2)(5a^2+60a+37)}{60(1-a)^2})+$$

$$g^{\alpha\beta}g^{\kappa\rho}Q_{\mu\alpha\beta}Q_{\nu\kappa\rho}(\frac{a(5a^2+a-1)}{12(1-a)}+$$

$$fraca(a-2)(5a^2+60a+37)-10a^2(a+3)60(1-a)^2)+$$

$$g^{\alpha\beta}g^{\kappa\rho}Q_{\mu\alpha\kappa}Q_{\nu\beta\rho}(\frac{a(5a^2+a-1)}{6(1-a)}+$$

$$\frac{a(a-2)(5a^2+60a+37)-10a^2(a+3)}{60(1-a)^2})+$$

$$g_{\mu\nu}g^{\alpha\beta}g^{\sigma\tau}Q_{\kappa\alpha\beta}Q^{\kappa}_{\sigma\tau}(-\frac{1}{6}+$$

$$\frac{5a(-5a^2-5a+8)}{120(1-a)^2}+\frac{a(a-2)(5a^2+45a+26)}{120(1-a)^2})+$$

$$g_{\mu\nu}g^{\alpha\beta}g^{\sigma\tau}Q_{\kappa\alpha\rho}Q^{\kappa}_{\beta\sigma}(-\frac{1}{3}+$$

$$\frac{5a(-5a^2-5a+8)+a(a-2)(5a^2+45a+26)}{60(1-a)^2})+$$

$$g_{\mu\nu}g^{\alpha\beta}Q^{\kappa}_{\alpha\rho}Q^{\rho}_{\kappa\beta}(-2+\frac{a(a-2)(5a^2+45a+26)}{30(1-a)^2})+g^{\alpha\beta}Q_{\mu\alpha\kappa}Q^{\kappa}_{\beta\nu}(\frac{-a(a-2)}{2}), \tag{23}$$

where $T^{\mu\nu}_{\alpha\beta}=\nabla^{\mu}\nabla^{nu}g_{\alpha\beta}$ and $Q_{\mu\alpha\beta}=\nabla_{mu}g_{\alpha\beta}$.

Here, we have calculated the lowest DWSG coefficients for the case of metrics incompatible with connection. We can encounter this problem in studying different unified models dealing with dynamically generated gravity. It has long been noted (Schrödinger, ref. [11]) that the basic concepts of the Einstein gravity (Riemann tensor, curvature, invariant differentiation and so on) are not characteristic for the models based on the metric connection. Instead, it is more natural to consider a models with only fundamental affine connection.

Another approach leading to metric incompatible with the connection is a hypothesis of the matter field as a source for the metric tensor. For instance, in [12,13] the usual Einstein action is obtained as a result of integration over quantum fluctuations of the fundamental matter (usually fermion) fields.

Metric tensor is then obtained as a vacuum background field. The compatibility of this field with the connection is viewed as dynamical equation (for example, the minimization of the vacuum energy leading to requirement for this background to be covariantly constant) rather than purely geometrical relation. Note that this approach can allow for a better ultraviolet behaviour.

At last, the metric incompatible with connection can appear in the effective theories when reducing the effective action to a purely quadratic form. Evidently, there is no reason why the (effective) metric tensor in such a case should automatically be compatible with a connection.

In work [14] it was suggested to find DWSG coefficients for operators in spaces with torsion by using DWSG coefficients for operators without torsion. This can be done redefinition of covariant derivative which again leads to the problem of calculation of DWSG coefficients in spaces with metric incompatible with connection.

4 Acknowledgments

The authors are grateful to V.P. Gusynin for many valuable remarks and fruitful discussions. The work was supported in part by the grants INTAS-93-2058-EXT "East-West network in constrained dynamical systems".

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